Impact of fermionic singlets on lepton universality tests

Cédric Weiland

Laboratoire de Physique Théorique d'Orsay, Université Paris-Sud 11, France

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Neutrino oscillations

• Oscillation probability is non-zero only if $\Delta m_{kj}^2 = m_k^2 - m_j^2$ and mixing angles are different from zero

$$P_{\nu_{\alpha} \to \nu_{\beta}} = \delta_{\alpha\beta} - 4 \sum_{k>j} \Re[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right)$$
$$+ 2 \sum_{k>j} \Im[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)$$

- Neutrino oscillations:
 - solar $\nu_e \to \nu_{\rm others}$: $\theta_{12} \simeq 33^\circ$, $\Delta m_{12}^2 \simeq 7.5 \times 10^{-5} {\rm eV}^2$ (best fit)
 - atmospheric $\stackrel{(-)}{\nu_{\mu}} \rightarrow \stackrel{(-)}{\nu_{\tau}}$: $\theta_{23} \simeq 40^{\circ} \text{ or } 50^{\circ}$, $|\Delta m_{23}^2| \simeq 2.4 \times 10^{-3} \text{eV}^2$ (best fit)
 - reactor $\bar{\nu}_e \rightarrow \bar{\nu}_{\text{others}}$: $\theta_{13} \simeq 8.7^{\circ}$ (best fit)
 - accelerator $\nu_{\mu} \rightarrow \nu_{\text{others}}$





Neutrino oscillations

Oscillations ⇒ Non-diagonal charged currents

$$\mathcal{L}_{\text{int}} = -\frac{g}{\sqrt{2}} \mathbf{U}^{ii}_{\nu} \bar{\ell}_{j} \gamma^{\mu} P_{L} \nu_{i} W_{\mu}^{-} + \text{h.c.}$$

• 3 mass eigenstates $\nu_i = \nu_1, \nu_2, \nu_3$ different from the interaction eigenstates $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$

$$\nu_{\alpha} = U_{\nu}^{\alpha i} \nu_{i}$$

- $\Rightarrow U_{\nu}$ is a 3 \times 3 unitary matrix, $U_{\nu} = U_{\rm PMNS}$
- U_{PMNS}: 3 mixing angles + 1 CP violating phase (+ 2 Majorana phases)
- Oscillations give no information on:
 - the absolute mass scale
 - the Dirac or Majorana nature of neutrinos





Neutrino masses and nature

- Absolute masses
 - Tritium β decays: $m_{\nu_e} < 2.05$ eV [Kraus et al., 2005, Aseev et al., 2011]
 - Cosmology: CMB $\Sigma m_{
 u_i} < 0.98$ eV Planck: [Ade et al., 2013] CMB+BAO+ H_0 +flat Universe $\Sigma m_{
 u_i} < 0.23$ eV
- Hierarchy: $m_1 < m_2 < m_3$ or $m_3 < m_1 < m_2$?
 - Matter effect in the Sun: ν_1 is mostly ν_e
 - $sign(\Delta m_{23}^2)$? \rightarrow matter effect in long baseline oscillations (T2K, NO ν A, future projects)
- Neutrino nature (Dirac or Majorana):
 - Neutrinoless double β decay: $m_{2\beta} < 0.12 0.25 \text{ eV}$ KamLAND-ZEN: [Gando et al., 2013]





Massive neutrinos and New Physics

- Standard Model $L = \binom{
 u_L}{\ell_L}, H = \binom{H^+}{H^0}$
 - No right-handed neutrino $\nu_R \to \text{No Dirac mass term}$

$$\mathcal{L}_{\text{mass}} = -Y_{\nu} \bar{L} \tilde{H} \nu_R + \text{h.c.}$$

No Higgs triplet T → No Majorana mass term

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}m\overline{L^C}TL + \text{h.c.}$$

- Necessary to go beyond the Standard Model for ν mass
 - Radiative models
 - Extra-dimensions
 - R-parity violation in supersymmetry
 - Seesaw mechanisms $\rightarrow \nu$ mass at tree-level
 - + BAU through leptogenesis





Dirac neutrinos?

• Add gauge singlet, right-handed neutrinos ν_R

$$\Rightarrow \nu = \nu_L + \nu_R$$

$$\mathcal{L}_{\text{mass}}^{\text{leptons}} = -Y_{\ell} \bar{L} H \ell_R - Y_{\nu} \bar{L} \tilde{H} \nu_R + \text{h.c.}$$

$$\Rightarrow \text{After electroweak symmetry breaking } \langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\mathcal{L}_{\text{mass}}^{\text{leptons}} = -v Y_{\ell} \bar{\ell}_{L} \ell_{R} - v Y_{\nu} \bar{\nu}_{L} \nu_{R} + \text{h.c.} = -m_{\ell} \bar{\ell}_{L} \ell_{R} - m_{D} \bar{\nu}_{L} \nu_{R} + \text{h.c.}$$

$$\Rightarrow$$
 3 light neutrinos: $m_{\nu} \le 1 \text{eV} \Rightarrow Y^{\nu} \le 10^{-11}$

Increase the hierarchy between Yukawa couplings





Majorana neutrinos?

- Add gauge singlet, right-handed neutrinos ν_R $\mathcal{L}_{\text{mass}}^{\text{leptons}} = -Y_{\ell}\bar{L}H\ell_R - Y_{\nu}\bar{L}\tilde{H}\nu_R - \frac{1}{2}M_R\overline{\nu_R^C}\nu_R + \text{h.c.}$
 - \Rightarrow After electroweak symmetry breaking $\langle H \rangle = \binom{0}{v}$

$$\mathcal{L}_{\text{mass}}^{\text{leptons}} = -m_{\ell} \ell_L \ell_R - m_D \bar{\nu}_L \nu_R - \frac{1}{2} M_R \overline{\nu_R^C} \nu_R + \text{h.c.}$$

- \Rightarrow 6 mass eigenstates: $\nu = \nu^C$
- ν_R gauge singlets
 - \Rightarrow M_R not related to SM dynamics, not protected by symmetries
 - $\Rightarrow M_R \overline{\nu_R^C} \nu_R$ is gauge and Lorentz invariant, renormalisable
- $M_R \overline{\nu_R^C} \nu_R$ violates lepton number conservation $\Delta L = 2$





The seesaw mechanism

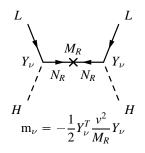
- $\Delta L = 2, m_{\nu} \neq 0 \Rightarrow \text{New physics}$
- Seesaw mechanism: New fields with a mass M_R > EW scale (in general) and Majorana mass terms
 ⇒ Generate m_v in a renormalizable way and at tree-level
- $\begin{array}{ll} \bullet \quad \text{Type I seesaw } \mathcal{L}_{\text{mass}}^{\text{leptons}} = -Y_\ell \bar{L} H \ell_R Y_\nu \bar{L} \tilde{H} \nu_R \frac{1}{2} M_R \overline{\nu_R^C} \nu_R + \text{h.c.} \\ \Rightarrow \text{After EWSB, neutrino mass matrix } M_0^{\epsilon} \times 6 \\ M^{\nu} = \begin{pmatrix} 0 & m_D \\ m_D^{\intercal} & m_R \end{pmatrix} & m_D = v Y_{\nu} \text{ Dirac mass matrix } \\ \Rightarrow \text{Seesaw limit } M_R \gg m_D & m_U^{\text{light}} \approx -m_D M_R^{-1} m_D^{\intercal} & \nu^{\text{light}} \approx \nu_L + \nu_L^C \\ m_{\nu}^{\text{heavy}} \approx M_R & \nu^{\text{heavy}} \approx \nu_R + \nu_R^C \\ \end{array}$
- Charged current matrix U_{ν} is 3×6
- 3 charged leptons + 3 light neutrinos: $\widetilde{U}_{\rm PMNS}$ = 3 × 3 submatrix of U_{ν} , maybe not unitary

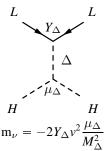


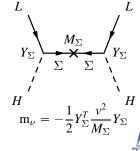


Three seesaw mechanisms

- Three minimal tree-level seesaw models ⇒ Three types of heavy fields
 - type I: right-handed neutrinos, SM gauge singlets
 - type II: scalar triplets
 - type III: fermionic triplets









Effective approach to seesaw mechanisms

- Notice that lepton number conservation is accidental in the SM (gauge group, field content and renormalizability)
- Unique dimension 5 operator for all seesaw mechanisms
 → Violates lepton number L ⇒ Majorana neutrinos

$$\delta \mathcal{L}^{d=5} = \frac{1}{2} c_{ij} \frac{(\bar{L}_i \tilde{H})^{\dagger} (\bar{L}_j \tilde{H})}{\Lambda} + \text{h.c.}$$

- To distinguish the several seesaw mechanisms, either
 - Directly produce the heavy states (LHC, LC)
 - Look for dimension ≥ 6 operator effects → charged lepton flavour violation (cLFV), non-standard interactions, etc





The inverse seesaw mechanism

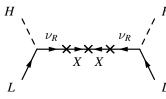
• Inverse seesaw \Rightarrow Consider fermionic gauge singlets ν_{Ri} $(L=+1,\,i=1,2,3)$ and X_i $(L=+1,\,i=1,2,3)$

$$\mathcal{L}_{inverse} = Y_{\nu}^{ij} \overline{L_i} \tilde{H} \nu_{Rj} - M_R^{ij} \overline{\nu_{Ri}} X_j - \frac{1}{2} \mu_X^{ij} \overline{X_i^C} X_j + \text{h.c.}$$

with
$$m_D = Y_{\nu} v$$
, $M^{\nu} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix}$

$$m_{
u} \approx \frac{m_D^2 \mu_X}{m_D^2 + M_R^2}$$

$$m_{1,2} \approx \mp \sqrt{m_D^2 + M_R^2} + \frac{M_R^2 \mu_X}{2(m_D^2 + M_R^2)}$$







The inverse seesaw mechanism

- Type I seesaw: $M_R \simeq 10^{15} \text{GeV}$ with natural Yukawa $Y_\nu \sim \mathcal{O}(1)$ or $M_R \sim 1 \text{TeV}$ with Yukawa $Y_\nu \sim \mathcal{O}(10^{-6})$
 - \Rightarrow Small active-sterile mixing $(\frac{m_D}{M_P}) \rightarrow \tilde{U}_{PMNS} \sim U_{PMNS}$
 - ⇒ Small deviations from unitarity
- Inverse seesaw: $M_R \simeq 1$ TeV with natural Yukawa $Y_{\nu} \sim \mathcal{O}(1)$
 - \Rightarrow Large active-sterile mixing $\rightarrow \tilde{U}_{PMNS} \neq U_{PMNS}$
 - ⇒ Large deviations from unitarity?
- Inverse seesaw: testable at the LHC and low energy experiments
 Could provide a sterile neutrino at the eV scale (reactor and LSND/MiniBooNE anomalies)





Lepton flavour universality

- Lepton flavour universality (LFU): gauge boson couplings are independent of lepton flavours
- Searches for LFU violation are among the most precise tests of the SM

$$\frac{\mathcal{B}(Z^0 \to \mu^+ \mu^-)}{\mathcal{B}(Z^0 \to e^+ e^-)} = 1.0009 \pm 0.0028$$
 [Schael et al., 2006]
$$\frac{\mathcal{B}(Z^0 \to \tau^+ \tau^-)}{\mathcal{B}(Z^0 \to e^+ e^-)} = 1.0019 \pm 0.0032$$

- Deviations from LFU ⇒ Evidence of New Physics
- Could help disentangle different neutrino mass generation mechanisms
 - Magnitude of LFU violation
 - Correlation between different LFU tests





Lepton universality tests

- Couplings to different bosons can be tested: γ, Z^0, W^{\pm} Impact of singlet neutrinos \Rightarrow Focus on W^{\pm} couplings
- Many observables can be used
 - Gauge boson decays (e.g. $W \to \ell \bar{\nu}$)
 - Leptonic and semileptonic meson decays (e.g. $K \to \ell \bar{\nu}, \overline{B} \to D \ell^- \bar{\nu}$)
 - Lepton decays (e.g. $\ell \to \ell' \nu \bar{\nu}$, $\tau \to K \nu$)
- We considered light meson decays: pions and kaons
 Decay width plagued by QCD uncertainties ⇒ Consider ratios

$$R_P = \frac{\Gamma(P^+ \to e^+ \nu)}{\Gamma(P^+ \to \mu^+ \nu)} \qquad P = K, \pi$$





Why R_K and R_{π} ?

Well measured by the NA62 collaboration [Lazzeroni et al., 2013]:

$$R_K^{\text{exp}} = (2.488 \pm 0.010) \times 10^{-5}$$

Current experimental error: $\frac{\delta R_K}{R_V} \simeq 0.4\%$

Expected sensitivity: $\frac{\delta R_K}{R_K} \simeq 0.1\%$

SM prediction is very precise [Finkemeier, 1996, Cirigliano and Rosell, 2007]:

$$R_K^{\text{SM}} = (2.477 \pm 0.001) \times 10^{-5}$$

• New Physics: $R_K = R_K^{SM} (1 + \Delta r_K)$

$$\Delta r_K = (4 \pm 4) \times 10^{-3}$$

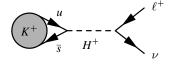
• Similar prospects for R_{π}





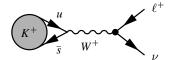
Deviations from the SM

- Origin of LFU violation in R_K:
 - New Lorentz structure in the four-fermion interaction



New fields, new couplings e.g. 2 Higgs doublet models, Supersymmetry

• Corrections to the SM $W\ell\nu$ vertex



New states, Higher-order effects e.g. Additional neutrinos: low-scale seesaw, inverse seesaw





New Lorentz structure

Two Higgs Doublet Model (2HDM) [Hou, 1993]

$$\Gamma^{\text{2HDM}}(K^+ \to \ell^+ \nu) = \Gamma^{\text{SM}}(K^+ \to \ell^+ \nu) \left(1 - \tan^2 \beta \frac{m_K^2}{m_{H^+}^2} \frac{m_s}{m_s + m_u} \right)^2$$

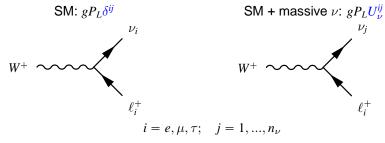
- \Rightarrow the tree-level correction is universal: $\Delta r_K^{2{\rm HDM}} \sim 0$
- Supersymmetry
 - Tree-level correction is identical to 2HDM
 → higher-order corrections are required [Masiero et al., 2006]
 - In the unconstrained MSSM $\mathcal{B}(B \to \tau \nu)$ limits $\Delta r_k^{ZHDM} < 10^{-3}$ [Fonseca et al., 2012]





Modified $W\ell\nu$ vertex

Naturally arises when leptonic mixing is added to the SM



- If $n_{\nu} = 3 \rightarrow U_{\nu} = U_{\rm PMNS}$, unitary
- If $n_{\nu} > 3$ (e.g. fermionic singlets) $\rightarrow U_{\nu} \neq U_{\rm PMNS}$ $\rightarrow U_{\nu}$ is a $3 \times n_{\nu}$ non-unitary matrix $\rightarrow \widetilde{U}_{\rm PMNS}$ is not unitary
- Tree-level corrections to R_K





Deviation from universality

• Summing over all the kinematically accessible neutrinos (from 1 to $N_{\max}^{(e)}$, $N_{\max}^{(\mu)}$ the heaviest kinematically allowed neutrino):

$$R_{K} = \frac{\sum_{i=1}^{N_{\max}^{(e)}} |U_{\nu}^{1i}|^{2} G^{i1}}{\sum_{k=1}^{N_{\max}^{(\mu)}} |U_{\nu}^{2k}|^{2} G^{k2}} \quad \text{with}$$

$$G^{ij} = \left[m_{K}^{2} (m_{\nu_{i}}^{2} + m_{l_{j}}^{2}) - (m_{\nu_{i}}^{2} - m_{l_{j}}^{2})^{2} \right] \left[(m_{K}^{2} - m_{l_{j}}^{2} - m_{\nu_{i}}^{2})^{2} - 4m_{l_{j}}^{2} m_{\nu_{i}}^{2} \right]^{1/2}$$

- In the SM + 3 massive ν , one recovers $R_K^{SM} = \frac{m_{\tilde{\tau}}^2 \; (m_K^2 m_{\tilde{\tau}}^2)^2}{m_{\mu}^2 \; (m_K^2 m_{\mu}^2)^2}$
 - $\bullet \ m_{\nu} \ll m_{\ell} \Rightarrow G^{i1} = G^{j1}$
 - $U_{\nu} = U_{\text{PMNS}} \Rightarrow \sum_{i=1}^{n_{\nu}} |U_{\nu}^{1i}|^2 = (U_{\nu}U_{\nu}^{\dagger})_{11} = 1$
- 2 ways to deviate from universality:
 - (A) sterile neutrinos are lighter than m_K , with $m_{\nu}^{\rm active} \ll m_{\nu_s} \lesssim m_K$ \to Phase space effect
 - (B) sterile neutrinos are heavier than the kaon, $m_{\nu_s} > m_K$ $\rightarrow \tilde{U}_{\rm PMNS}$ is not unitary





Is there a visible deviation from LFU?

Yes!

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- Effect of deviation from unitarity [Shrock, 1980, 1981]
- However, current experimental constraints on sterile neutrinos and non-unitary are quite stringent





Constraints on sterile neutrinos

Depend on the mass regime and the size of active-sterile mixing

- Direct searches (e.g. monochromatic lines in $\pi \to \mu \nu$): [Atre et al., 2009, Kusenko, 2009]
- Non-unitarity constraint $\widetilde{U}_{\mathrm{PMNS}} = (1 \eta) U_{\mathrm{PMNS}}$: [Antusch et al., 2009]
- Lepton flavour violation (e.g. $\mu \rightarrow e\gamma$): [Deppisch and Valle, 2005]
- B Physics (e.g. $B \rightarrow \ell \nu$)





Constraints on sterile neutrinos

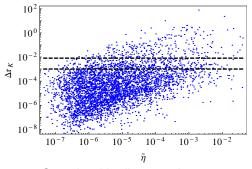
- Depend on the mass regime and the size of active-sterile mixing
 - LHC Higgs searches (e.g. invisible decays):
 [Bhupal Dev et al., 2012, Cely et al., 2013]
 - Electroweak precision data: [del Aguila et al., 2008, Atre et al., 2009]
 - Cosmological observations (e.g. LSS, Lyman-α, CMB, BBN, X-ray): [Smirnov and Zukanovich Funchal, 2006, Kusenko, 2009]
 → can be evaded with non-standard cosmology (e.g. low reheating temperature [Gelmini et al., 2008])





R_K in the inverse seesaw

- Inverse seesaw as an illustrative example, only one among other possibilities
- Numerical results in scenario (A): $m_{\nu}^{\rm active} \ll m_{\nu_s} \lesssim m_K$



 $M_R \in [0.1 \text{ MeV}, 200 \text{ MeV}]$ $\mu_X \in [0.01 \text{ eV}, 1 \text{ MeV}]$

$$\tilde{\eta} = 1 - |\mathsf{Det}(\tilde{U}_{\mathsf{PMNS}})|$$

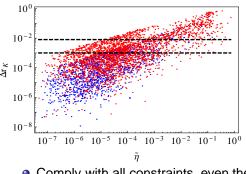
- Comply with all constraints, except cosmological bounds
- Large LFU violation $\Delta r_K \sim 1$ can be reached
- Small $Y_{\nu} \rightarrow$ no cLFV signal expected





• Numerical results in scenario (B): $m_{\nu_s} > m_K$

Lepton Universality



$$M_R \in [1 \text{ GeV}, 10^6 \text{ GeV}]$$

 $\mu_X \in [0.01 \text{ eV}, 1 \text{ MeV}]$
 $Y_{\nu} > 10^{-2}$ $Y_{\nu} < 10^{-2}$

$$\tilde{\eta} = 1 - |\mathsf{Det}(\tilde{U}_{\mathsf{PMNS}})|$$

- Comply with all constraints, even the stringent non-unitarity bounds
- Large LFU violation $\Delta r_K \sim 1$ can be reached
- Large $Y_{\nu} \to \mathcal{B}(\mu \to e\gamma)$ is within MEG reach
- Specific to the inverse seesaw with its large active-sterile mixing





R_K summary

- Source: modified $W\ell\nu$ vertex from extra sterile neutrinos
- Mechanism: phase space effect for $m_{
 u}^{
 m active} \ll m_{
 u_s} \lesssim m_K$ non-unitarity of $\widetilde{U}_{
 m PMNS}$ for $m_{
 u_s} > m_K$
- Results: large LFU violation ($\Delta r_K \lesssim 1$) in the inverse seesaw
- Similar results for R_{π}





• Leptonic W^{\pm} decays

$$R_{\tau\ell}^W = \frac{2\mathcal{B}(W \to \tau \bar{\nu}_{\tau})}{\mathcal{B}(W \to \mu \bar{\nu}_{\mu}) + \mathcal{B}(W \to e\bar{\nu}_{e})} = 1.077 \pm 0.026$$

 \Rightarrow 2.8 σ deviation from the SM prediction $R_{\tau\ell}^W|_{SM}=0.999$ [LEP, 2005, Kniehl et al., 2000]

Other leptonic meson decays

$$R_{D_s} = \frac{\Gamma(D_s^+ \to \tau^+ \nu)}{\Gamma(D_s^+ \to \mu^+ \nu)} \simeq 9.2$$

 \Rightarrow roughly 1σ away from the SM prediction $R_{D_S}|_{SM} \simeq 10.1$ [Beringer et al., 2012, Charles et al., 2011]



• Semileptonic τ decays

$$R_{K,\mu}^{\tau} = \frac{\mathcal{B}(\tau^- \to K^- \nu)}{\mathcal{B}(K^+ \to \mu^+ \nu)} \simeq 469$$

 \Rightarrow within 1σ of the SM prediction $R_{K,\mu}|_{SM} \simeq 474$ [Beringer et al., 2012]

Semileptonic meson decays

$$R(D) = \frac{\mathcal{B}(\overline{B} \to D\tau^- \bar{\nu}_{\tau})}{\mathcal{B}(\overline{B} \to D\ell^- \bar{\nu}_{\ell})} = 0.440 \pm 0.072$$

 \Rightarrow deviates by 1.7 σ from the SM prediction $R(D)|_{SM}=0.31\pm0.02$ [Lees et al., 2012, Becirevic et al., 2012]



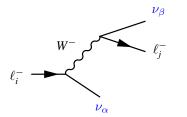
- 3-body lepton decays: $\ell_i \to \ell_i \nu \nu$
- Test LFU via ratios, e.g.

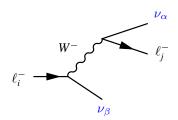
$$R_{\tau} = \frac{\Gamma(\tau^- \to \mu^- \nu \nu)}{\Gamma(\tau^- \to e^- \nu \nu)} = 0.9764 \pm 0.0030$$

- \Rightarrow within 2σ of the SM prediction $R_{\tau}|_{SM} \simeq 0.9726$ [Beringer et al., 2012]
- 2 massive neutrinos in the final state
 - → Sensitive to the Majorana or Dirac nature of the neutrino









• $\Gamma_{\text{Majorana}} = \Gamma_{\text{Dirac}} + \Gamma_{\text{interferences}}$

$$\Gamma_{\mathrm{interferences}} \propto \Re(U_{i\alpha}^* U_{j\beta} U_{i\beta} U_{j\beta}^*) \frac{m_{\nu_{\alpha}} m_{\nu_{\beta}}}{m_{\ell_i}^2}$$

• Large active-sterile mixing + heavy sterile neutrinos $(m_{\nu_s} \sim \mathcal{O}(m_{\ell_i}))$ \Rightarrow Potentially sizeable deviations (under investigation)



Conclusion

- Lepton universality tests: good theoretical precision and experimental prospects
- Sterile neutrinos can lead to a large violation of LFU at tree-level
- R_K particularly well-suited for this search
- Large deviations from the SM can be found
- Can appear in other observables with leptonic charged currents
 - → Currently under investigation





Backup slides





Leptogenesis

- Generate the baryonic (leptonic) asymmetry → Sakharov conditions [Sakharov, 1967]
 - Out of equilibrium process
 - Baryon (lepton) number violation
 - C and CP violation
- Impossible in the Standard Model: not enough CP violation [Gavela et al., 1994]
- Use the leptonic sector
 - Majorana mass term violates lepton number conservation \Rightarrow Passed to the baryonic sector via sphalerons (B-L conserving)
 - Neutrinos mass matrix \Rightarrow Extra sources of CP violation (δ_{13} , $\alpha_{1,2}$)





Scan method

- Inverse seesaw as an illustrative example, only one among other possibilities
- Random scan on M_R and μ_X entries
- Y_{ν} obtained from neutrino data via the Casas-Ibarra parametrization [Casas and Ibarra, 2001]

$$Y_{\nu}^{T} = \frac{\sqrt{2}}{\nu} V^{\dagger} \operatorname{diag}(\sqrt{m_{1}}, \sqrt{m_{2}}, \sqrt{m_{3}}) R \operatorname{diag}(\sqrt{M_{1}}, \sqrt{M_{2}}, \sqrt{M_{3}}) U_{PMNS}^{\dagger}$$

where R is a complex orthogonal matrix and V a unitary matrix that decompose $M = M_R \mu_X^{-1} M_R^T$ according to $M = V^{\dagger} \operatorname{diag}(M_1, M_2, M_3) V^*$.

Apply the constrains



